Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **22** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y]=\frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	**B1**	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
	$5=\frac{4}{-2}(4-3)^{-2}+c$ or $5=\frac{4}{-2(4-3)^2}+c$ leading to $c=$	**M1**	Correct use of $(4,5)$ to find c in an integrated expression (defined by the correct power and no extra x's or terms).
	$y=\frac{-2}{(x-3)^2}+7$ or $y=-2(x-3)^{-2}+7$	**A1**	OE $-\frac{4}{2}$ must be simplified to –2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
		3	

Question	Answer	Marks	Guidance
2	[Coefficient of $x^4 = p =$] $15a^2$	**B1**	May be seen in an expansion or with x^4.
	[Coefficient of $x^2 = q =$] $54a^2$	**B1**	May be seen in an expansion or with x^2.
	Equating *their p* + *their q* to 276 leading to an equation in a^2 only	**M1**	No x terms and no extra terms. If p and q are not identified then it needs to be clear from the expansion that the appropriate coefficients are being used. $69a^2=276$ implies the first 3 marks.
	$a=\pm 2$	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
3(a)	$4(x-3)^2$ seen or $a = 4$ and $b = -3$	**B1**	OE Award marks for the correct expression or their values *a*, *b* and *c*. Condone $4(x-3) + p - 36 = 0$ and $4\left(\frac{p}{4} - 9\right)$.
	$-36 + p$ or $p - 36$ seen or $c = p - 36$	**B1**	
		2	
3(b)	$p - 36 > 0$ leading to $p > 36$ or $24^2 - 4 \times 4p \langle 0 \Rightarrow p \rangle 36$ or $36 < p$	**B1**	Allow $(36, \infty)$ or $36 < p < \infty$. Consider final answer only.
		1	

Question	Answer	Marks	Guidance
4	$[8x^6 + 215x^3 - 27 = 0]$ leading to $(8x^3 - 1)(x^3 + 27)[= 0]$ **OR** $\frac{-215 \pm \sqrt{215^2 - 4.8.-27}}{2.8}$ or $\frac{-215 \pm \sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x = x^3$.
	$\frac{1}{8}$, -27	**A1**	Both correct values seen. **SC**: if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE
	$\frac{1}{2}$ or $0.5, -3$	**A1**	**SC**: if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
		3	

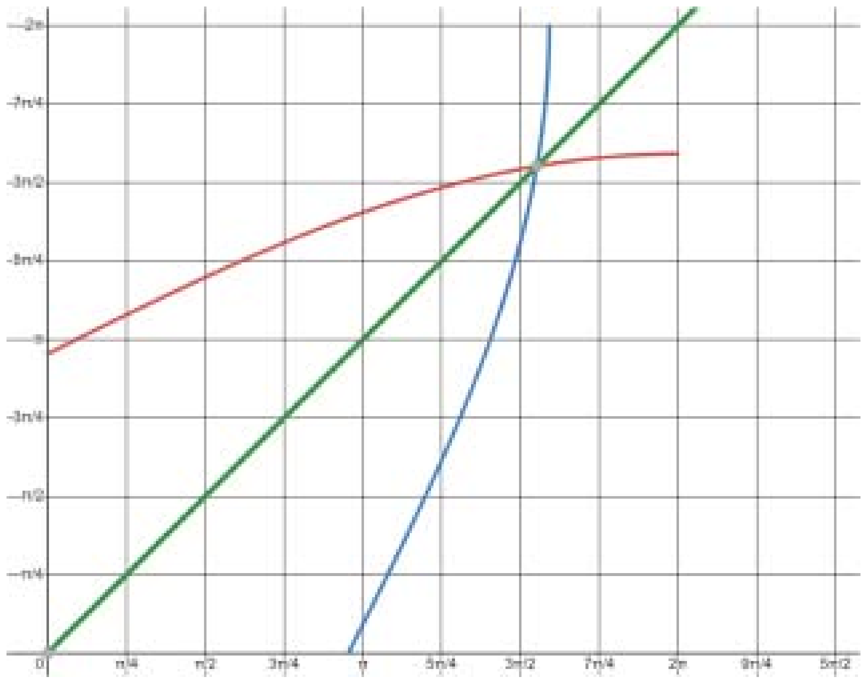
Question	Answer	Marks	Guidance
5	[$\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)$ =] $\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}$ $\left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	**B1 B1**	B1 for contents of each { } then ISW.
	$=\left(\textit{their}\,\frac{20}{3}\times 8-32\right)[-0]$	**M1**	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their ($\frac{160}{3}-32$).
	[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	**A1**	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
		4	

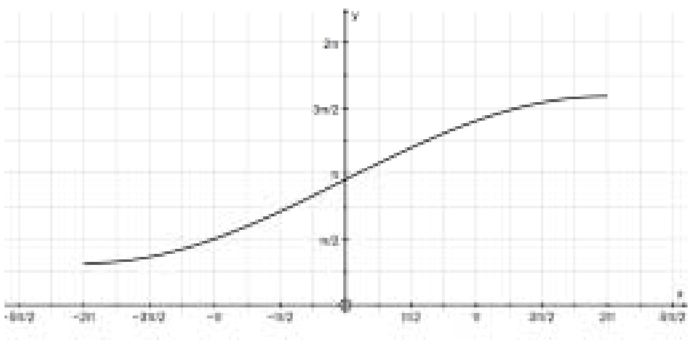
Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi - 2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi - 2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	***B1**	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow *p*, *a* or *r* for *OA*) containing only terms with *x* and θ but not just $OA = 2x\cos\theta$. Do not condone $\sin\pi - 2\theta$ until missing brackets recovered or $\cos(180 - 2\theta)$ until it becomes $-cos 2\theta$ etc.
	$OA = 2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$	**DB1**	AG Complete correct method showing all necessary working. Condone $2x\cos\theta \times \theta$.
		2	If B0 but www then **SCB1** for $OA = 2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$.
6(b)	Sector area = $\frac{1}{2}(2x\cos\theta)^2 \times \theta$	**M1**	OE Using sector formula with a correct OA. Condone $cos\theta^2$ for $cos^2\theta$ and missing brackets.
	Triangle area = $\frac{1}{2} \times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi - 2\theta)$	**M1**	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π.
	[Area *APB* =] *Their* sector area – *their* triangle area	**M1**	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area *APB* =] $\frac{1}{2}(2x\cos\theta)^2 \times \theta - \frac{1}{2}x^2\sin(\pi - 2\theta)$ [$= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)$]	**A1**	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect ‘simplifications’.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$	***B1**	Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www.
	$[\theta =]\ 0,\ \frac{\pi}{2},\pi$	**DB 2,1,0**	B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. **SC DB1** for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents.
		3	Verifying 3 answers rather than expanding and solving 0/3.
7(a)(ii)	$\cos 0 + \sin 0 = [1 + 0 =]1$ and $\cos\frac{\pi}{2} + \sin\frac{\pi}{2}[= 0 + 1] = 1$	**B1**	Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees.
	$\cos\pi + \sin\pi[= -1 + 0] = -1$ or $\neq 1$	**B1**	www
		2	

Question	Answer	Marks	Guidance
7(b)	$\dfrac{(\cos\theta-\sin\theta)\sin\theta+(\cos\theta+\sin\theta)(1-\cos\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$	**M1**	Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta-\sin^2\theta$. Condone brackets missing if recovered.
	$=\dfrac{\cos\theta\sin\theta-\sin^2\theta+\cos\theta-\cos^2\theta+\sin\theta\ -\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$	**A1**	
	$=\dfrac{\sin\theta+\cos\theta-\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}=\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$	**A1**	AG Clear evidence of using $sin^2\theta+cos^2\theta=1$ in either the numerator or denominator. Condone c, s and/or omission of θ . Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1.
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$ leading to $1=2(1-2\sin^2\theta)$	*M1	Replacing LHS with the expression from **(b)** and attempting to simplify i.e. condone omission of $(\cos\theta+\sin\theta-1)=0$ at this stage. M0 for $0=2(1-2\sin^2\theta)$
	$k\sin^2\theta=1$ or 3 leading to $\sin\theta=[\pm]\sqrt{\dfrac{1\text{ or }3}{k}}$ $\left[4sin^2\theta=1 \text{ leading to } \sin\theta=\pm\dfrac{1}{2}\right]$	DM1	Dividing by k and taking the square root of a positive value < 1. This mark can be implied by the solutions $\dfrac{1}{6}\pi, \dfrac{5}{6}\pi$.
	Solutions $0, \dfrac{1}{6}\pi, \dfrac{1}{2}\pi, \dfrac{5}{6}\pi$	A1	Allow 0, 0.524, 1.57, 2.62 AWRT. If M0 **SCB1** for $(\cos\theta+\sin\theta-1)=0 \Rightarrow 0, \dfrac{1}{2}\pi$. If M0 **SCB1** for all four correct answers and no others. Ignore answers outside of the range. Answers in degrees A0.
		3	

Question	Answer	Marks	Guidance
8(a)		***B1**	The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $\mathrm{f}^{-1}(x)$.
		DB1	Fully correct (needs to reach $y = 2\pi$ and x-axis and cross the line $y = x$ in the correct squares).
		2	

Question	Answer	Marks	Guidance
8(b)	$y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$	**M1**	Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone $\pm$ sign errors. Variables may be interchanged initially. M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$.
	$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to [$\mathrm{f}^{-1}(x)$ or y =] $4\sin^{-1}\left(\frac{x-3}{2}\right)$	**A1**	ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$. Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$.
		2	
8(c)		**B1**	Continuing given graph from y intercept to –2π. The correct shape needed between 0 and -2π, including starting to level off (gradient in the final two squares needs to be reducing) as -2π is approached. The y co-ordinate at-2π must be in the correct square.
	Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function].	**B1 FT**	If there is no graph to the left of the y axis, no mark is available. FT an incorrect graph and if the answer is now 'No' provide an appropriate reason.
		2	

Question	Answer	Marks	Guidance
8(d)	**{ } indicates different elements throughout.**		
	{Stretch} {factor 4} {in x-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of x-axis, horizontally or y-axis invariant. Wavelength or period increased by a factor of 4 for B2 or by 4 for B1.
	{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of y-axis, vertically or x-axis invariant. Allow y 'co-ordinates' doubled or amplitude doubled for B2.
	{Translation} $\begin{pmatrix} \{0\} \\ \{3\} \end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Allow shift. Any mention of y axis, y-direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1.
		6	After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1.

Question	Answer	Marks	Guidance
9(a)	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $a = \dfrac{16}{r}$ and $a = 100(1-r)$	**B1**	Rearranging two algebraic statements to give $a =$. These can be implied by a correct equation in one variable.
	$100(1-r)r = 16$ leading to $100r^2 - 100r + 16[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1) = 0$ OR $\dfrac{25 \pm \sqrt{25^2 - 4.25.4}}{2.25}$ leading to $r = \left[\dfrac{4}{5} \text{ or } \dfrac{1}{5}\right]$	**DM1**	Condone $(5r-4)(5r-1)$ following $100r^2 - 100r + 16$.
	a = 20, *a* = 80	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $r = \dfrac{16}{a}$ and $r = \dfrac{100-a}{100}$	**B1**	Rearranging two algebraic statements to give $r =$. These can be implied by a correct equation in one variable.
	$1600 = 100a - a^2$ leading to $a^2 - 100a + 1600[=0]$	***M1**	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80) = 0$ OR $\dfrac{100 \pm \sqrt{100^2 - 4.1600}}{2}$	**DM1**	
	a = 20, *a* = 80	**A1**	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
		4	

Question	Answer	Marks	Guidance
9(b)	$r = \frac{4}{5}\ ,\ \frac{1}{5}$	**B1**	OE SOI
	$[u_n =] their\ 20 \times their\left(\frac{4}{5}\right)^{n-1}\quad [v_n =] their 80 \times their\left(\frac{1}{5}\right)^{n-1}$	**B1FT**	2 expressions for the nth term FT *their* values from part **(a)** if $\|r\|$ less than 1.
	Method 1 for final 2 marks		
	$20 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1}$	**M1**	Correctly separating the numerator and denominator of *their* $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n = 80 \times \frac{4^{n-2}}{5^{n-1}}$.
	$u_n = \frac{1}{4} \times 80 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1} = 4^{n-2} \times 80 \times \left(\frac{1}{5}\right)^{n-1} = 4^{n-2} \times v_n$	**A1**	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20 \times 0.8^{n-1}}{80 \times 0.2^{n-1}} = \frac{1}{4} \times 4^{n-1}$	**M1**	Dividing two nth terms of the correct format and simplifying their terms in *r*.
	$= 4^{-1} \times 4^{n-1} = 4^{n-2}$	**A1**	AG
		4	

Question	Answer	Marks	Guidance
10(a)	$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$ or using $x=2y-12$	***M1**	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2+(3-2a)x+a^2-11[=0]$	**A1**	OE e.g. $5x^2+4(3-2a)x+4a^2-44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2-y(54+4a)+133+a^2+24$.
	$(3-2a)^2-4\times\frac{5}{4}(a^2-11)[=0]$	**DM1**	OE Using b^2-4ac on *their* 3 term quadratic $[=0]$.
	Method 1 for final 2 marks		
	Using a = 4: $(3-8)^2-5(5)=0$	**A1**	Clearly substituting a = 4.
	$a=-16$	**B1**	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2-12a+64=0 \Rightarrow (a-4)(a+16)=0 \Rightarrow a=4$	**A1**	AG Full method clearly shown.
	$a=-16$	**B1**	Condone no method shown for this value.
		5	If M0, **SCB1** available for substituting $a=4$, finding P(2, 7) and verifying that CP^2 = 20.

Question	Answer	Marks	Guidance
10(b)	Centre (4, 3) identified or used or the point P is (2, 7)	**B1**	
	$\therefore$ gradient of normal $= -2$	**B1**	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	**M1**	Condone use of $(\pm 4, \pm 3)$.
	$\frac{y-3}{(x-4)} = -2$ or $y - 7 = -2(x-2)$	**A1**	OE Condone $\mathrm{f}(x) =$.
		4	

Question	Answer	Marks	Guidance
10(c)	**Method 1 for Question 10(c)**		
	Diameter: $y-3=\frac{1}{2}(x-4)$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$ Or $2(x-4)+2(y-3)\frac{dy}{dx}=0$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$	***M1**	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$ $\left[\frac{5}{4}x^2-10x=0\right]$	**DM1**	Obtaining an unsimplified equation in *x* or *y* only. $[y^2-6y+5=0]$.
	x = 0 or 8, *y* = 1 or 5 [(0, 1) and (8, 5)]	**A1**	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y-1=-2x$ and $y-5=-2(x-8)$	**A1**	$2x+y=1$ and $2x+y=21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are (4+4, 3+2) = (8, 5) and (4 – 4, 3 – 2) = (0, 1)	***M1 A1**	Vector approach using their centre and gradient = 0.5 . Condone answers only with no working.
	Equations are $y-5=-2(x-8)$ and $y-1=-2x$	**DM1 A1**	Forming equations of tangents using *their* (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x-4)^2+(-2x+c-3)^2=20$ $\left[5x^2+(4-4c)x+(c-3)^2-4=0\right]$	***M1**	Obtaining an unsimplified equation in *x* only using equation of circle with $y=-2x+c$.
	$(4-4c)^2-20\left((c-3)^2-4\right)[=0]$ [leading to $-4c^2-32c+120c+16-100=0$]	**DM1**	Using $b^2-4ac[=0]$.

Question	Answer	Marks	Guidance
10(c)	$4c^2 - 88c + 84[=0]$ [leading to $c^2 - 22c + 21 = 0$]	A1	
	$c = 21$ and $c = 1$ or $y = -2x + 21$ **and** $y = -2x + 1$	A1	Condone no method shown for solution.
		4	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{k\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right\}\{\times 4\}\{-1\}$	B 2,1,0	OE e.g. $2k(4x+1)^{-\frac{1}{2}} - 1$ B2 Three correct unsimplified **{ }** and no others. B1 Two correct **{ }** or three correct **{ }** and an additional term e.g. +5. B0 More than one error.
		2	
11(b)	$2k(4x+1)^{-\frac{1}{2}} - 1 = 0$ leading to $(4x+1)^{\frac{1}{2}} = 2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}} = 1$	M1	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}} - 1$ where a = 2 or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x = \frac{4k^2 - 1}{4}$	A1	CAO OE simplified expression ISW.
		2	

Question	Answer	Marks	Guidance
11(c)	$2\times 10.5(4x+1)^{-\frac{1}{2}}-1=2$	**M1**	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7=(4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	**A1**	If M1 earned **SCB1** available for $x=\frac{33}{64}$ from $a=\frac{1}{2}$.
	$y=\left[10.5\sqrt{4x+1}-x+5=\right]66.5$ [leading to (12, 66.5)]	**A1**	
	$y-66.5=-\frac{1}{2}(x-12)$	**A1**	OE
		4	